7.5.1 Exercises

For a link to all of the additional resources available for this section, click OSttS Chapter 7 materials.

In Exercises 1 - 8, graph the hyperbola. Find the center, the lines which contain the transverse and conjugate axes, the vertices, the foci and the equations of the asymptotes.

For help with these exercises click on the resource below:

- Graphing hyperbolas

1. $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$
2. $\dfrac{y^2}{9} - \dfrac{x^2}{16} = 1$
3. $\dfrac{(x-2)^2}{4} - \dfrac{(y+3)^2}{9} = 1$
4. $\dfrac{(y-3)^2}{11} - \dfrac{(x-1)^2}{10} = 1$
5. $\dfrac{(x+4)^2}{16} - \dfrac{(y-4)^2}{1} = 1$
6. $\dfrac{(x+1)^2}{9} - \dfrac{(y-3)^2}{4} = 1$
7. $\dfrac{(y+2)^2}{16} - \dfrac{(x-5)^2}{20} = 1$
8. $\dfrac{(x-4)^2}{8} - \dfrac{(y-2)^2}{18} = 1$

In Exercises 9 - 12, put the equation in standard form. Find the center, the lines which contain the transverse and conjugate axes, the vertices, the foci and the equations of the asymptotes.

For help with these exercises click on the resource below:

- Completing the square to put the equation of a conic section in standard form

9. $12x^2 - 3y^2 + 30y - 111 = 0$
10. $18y^2 - 5x^2 + 72y + 30x - 63 = 0$
11. $9x^2 - 25y^2 - 54x - 50y - 169 = 0$
12. $-6x^2 + 5y^2 - 24x + 40y + 26 = 0$

In Exercises 13 - 18, find the standard form of the equation of the hyperbola which has the given properties.

For help with these exercises click on the resource below:

- Finding the equation of a hyperbola

13. Center $(3, 7)$, Vertex $(3, 3)$, Focus $(3, 2)$
14. Vertex $(0, 1)$, Vertex $(8, 1)$, Focus $(-3, 1)$
15. Foci $(0, \pm 8)$, Vertices $(0, \pm 5)$.
16. Foci $(\pm 5, 0)$, length of the Conjugate Axis 6
17. Vertices $(3, 2)$, $(13, 2)$; Endpoints of the Conjugate Axis $(8, 4)$, $(8, 0)$
18. Vertex $(-10, 5)$, Asymptotes $y = \pm\frac{1}{2}(x-6) + 5$

In Exercises 19 - 28, find the standard form of the equation using the guidelines on page 564 and then graph the conic section.

For help with these exercises click on one of the resources below:

- Completing the square to put the equation of a conic section in standard form
- Graphing a circle whose equation is in standard form
- Graphing parabolas
- Graphing ellipses
- Graphing hyperbolas

19. $x^2 - 2x - 4y - 11 = 0$
20. $x^2 + y^2 - 8x + 4y + 11 = 0$
21. $9x^2 + 4y^2 - 36x + 24y + 36 = 0$
22. $9x^2 - 4y^2 - 36x - 24y - 36 = 0$
23. $y^2 + 8y - 4x + 16 = 0$
24. $4x^2 + y^2 - 8x + 4 = 0$
25. $4x^2 + 9y^2 - 8x + 54y + 49 = 0$
26. $x^2 + y^2 - 6x + 4y + 14 = 0$
27. $2x^2 + 4y^2 + 12x - 8y + 25 = 0$
28. $4x^2 - 5y^2 - 40x - 20y + 160 = 0$
29. The graph of a vertical or horizontal hyperbola clearly fails the Vertical Line Test, Theorem 1.1, so the equation of a vertical of horizontal hyperbola does not define y as a function of x.[8] However, much like with circles, horizontal parabolas and ellipses, we can split a hyperbola into pieces, each of which would indeed represent y as a function of x. With the help of your classmates, use your calculator to graph the hyperbolas given in Exercises 1 - 8 above. How many pieces do you need for a vertical hyperbola? How many for a horizontal hyperbola?
30. The location of an earthquake's epicenter − the point on the surface of the Earth directly above where the earthquake actually occurred − can be determined by a process similar to how we located Sasquatch in Example 7.5.5. (As we said back in Exercise 75 in Section 6.1, earthquakes are complicated events and it is not our intent to provide a complete discussion of the science involved in them. Instead, we refer the interested reader to a course in Geology or the U.S. Geological Survey's Earthquake Hazards Program found here.) Our technique works only for relatively small distances because we need to assume that the Earth is flat in order to use hyperbolas in the plane.[9] The P-waves ("P" stands for Primary) of an earthquake in Sasquatchia travel at 6 kilometers per second.[10] Station A records the waves first. Then Station B, which is 100 kilometers due north of Station A, records the waves 2 seconds later. Station C, which is 150 kilometers due west of Station A records the waves 3 seconds after that (a total of 5 seconds after Station A). Where is the epicenter?

[8]We will see later in the text that the graphs of certain rotated hyperbolas pass the Vertical Line Test.

[9]Back in the Exercises in Section 1.1 you were asked to research people who believe the world is flat. What did you discover?

[10]Depending on the composition of the crust at a specific location, P-waves can travel between 5 kps and 8 kps.

31. The notion of eccentricity introduced for ellipses in Definition 7.5 in Section 7.4 is the same for hyperbolas in that we can define the eccentricity e of a hyperbola as

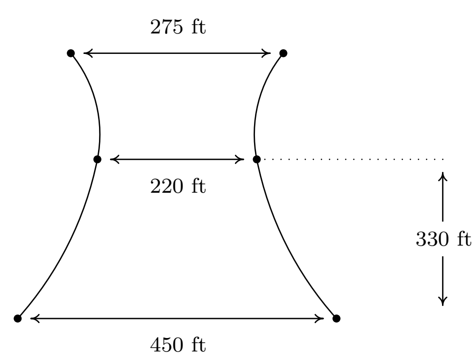
$$e = \frac{\text{distance from the center to a focus}}{\text{distance from the center to a vertex}}$$

   (a) With the help of your classmates, explain why $e > 1$ for any hyperbola.

   (b) Find the equation of the hyperbola with vertices $(\pm 3, 0)$ and eccentricity $e = 2$.

   (c) With the help of your classmates, find the eccentricity of each of the hyperbolas in Exercises 1 - 8. What role does eccentricity play in the shape of the graphs?

32. On page 533 in Section 7.3, we discussed paraboloids of revolution when studying the design of satellite dishes and parabolic mirrors. In much the same way, 'natural draft' cooling towers are often shaped as **hyperboloids of revolution**. Each vertical cross section of these towers is a hyperbola. Suppose the a natural draft cooling tower has the cross section below. Suppose the tower is 450 feet wide at the base, 275 feet wide at the top, and 220 feet at its narrowest point (which occurs 330 feet above the ground.) Determine the height of the tower to the nearest foot.



33. With the help of your classmates, research the Cassegrain Telescope. It uses the reflective property of the hyperbola as well as that of the parabola to make an ingenious telescope.

34. With the help of your classmates show that if $Ax^2 + Cy^2 + Dx + Ey + F = 0$ determines a non-degenerate conic[11] then

   - $AC < 0$ means that the graph is a hyperbola
   - $AC = 0$ means that the graph is a parabola
   - $AC > 0$ means that the graph is an ellipse or circle

   NOTE: This result will be generalized in Theorem 11.11 in Section 11.6.1.

[11] Recall that this means its graph is either a circle, parabola, ellipse or hyperbola.

Checkpoint Quiz 7.5

1. Put $25x^2 - 50x - 4y^2 - 16y - 91 = 0$ into standard form and graph. Find the center, vertices, foci, and asymptotes.

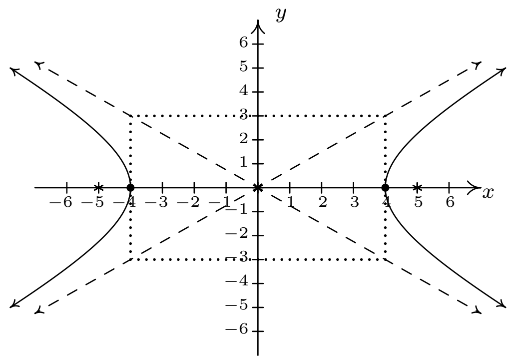
2. Find the equation of the hyperbola with vertices $(0, 0)$ and $(0, 4)$ and a focus at $(0, -1)$.

For worked out solutions to this quiz, click the links below:

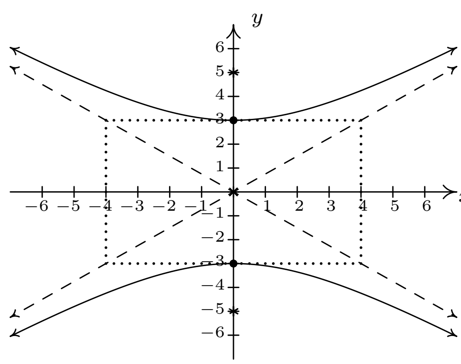
- Quiz Solution Part 1
- Quiz Solution Part 2

7.5.2 Answers

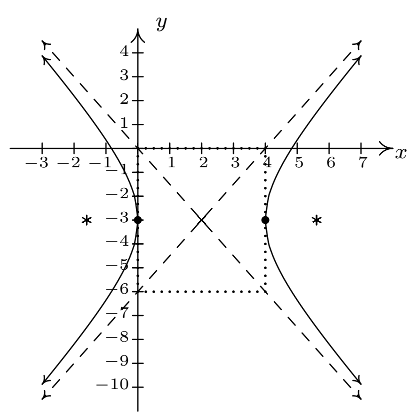
1. $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$
   Center $(0,0)$
   Transverse axis on $y = 0$
   Conjugate axis on $x = 0$
   Vertices $(4,0), (-4,0)$
   Foci $(5,0), (-5,0)$
   Asymptotes $y = \pm\frac{3}{4}x$



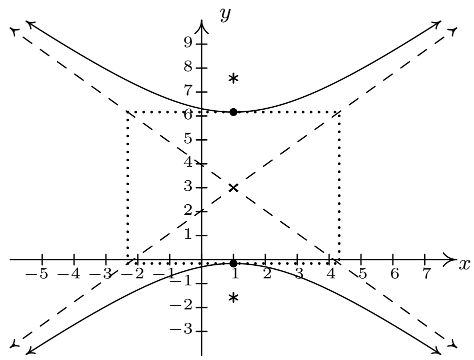
2. $\dfrac{y^2}{9} - \dfrac{x^2}{16} = 1$
   Center $(0,0)$
   Transverse axis on $x = 0$
   Conjugate axis on $y = 0$
   Vertices $(0,3), (0,-3)$
   Foci $(0,5), (0,-5)$
   Asymptotes $y = \pm\frac{3}{4}x$



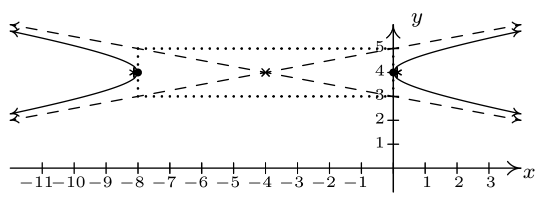
3. $\dfrac{(x-2)^2}{4} - \dfrac{(y+3)^2}{9} = 1$
   Center $(2,-3)$
   Transverse axis on $y = -3$
   Conjugate axis on $x = 2$
   Vertices $(0,-3), (4,-3)$
   Foci $(2+\sqrt{13},-3), (2-\sqrt{13},-3)$
   Asymptotes $y = \pm\frac{3}{2}(x-2) - 3$



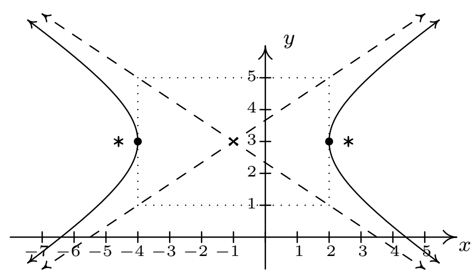
4. $\dfrac{(y-3)^2}{11} - \dfrac{(x-1)^2}{10} = 1$
Center $(1,3)$
Transverse axis on $x = 1$
Conjugate axis on $y = 3$
Vertices $(1, 3+\sqrt{11}), (1, 3-\sqrt{11})$
Foci $(1, 3+\sqrt{21}), (1, 3-\sqrt{21})$
Asymptotes $y = \pm\frac{\sqrt{110}}{10}(x-1)+3$



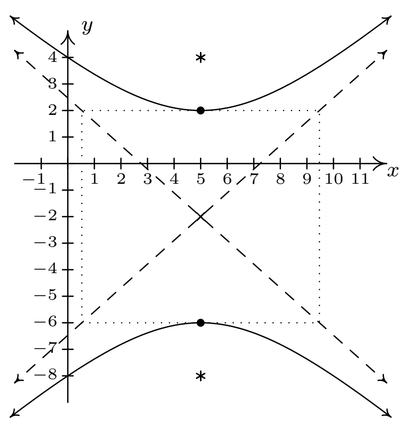
5. $\dfrac{(x+4)^2}{16} - \dfrac{(y-4)^2}{1} = 1$
Center $(-4,4)$
Transverse axis on $y = 4$
Conjugate axis on $x = -4$
Vertices $(-8,4), (0,4)$
Foci $(-4+\sqrt{17}, 4), (-4-\sqrt{17}, 4)$
Asymptotes $y = \pm\frac{1}{4}(x+4)+4$



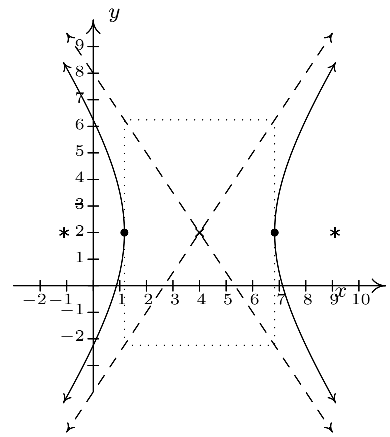
6. $\dfrac{(x+1)^2}{9} - \dfrac{(y-3)^2}{4} = 1$
Center $(-1,3)$
Transverse axis on $y = 3$
Conjugate axis on $x = -1$
Vertices $(2,3), (-4,3)$
Foci $\left(-1+\sqrt{13}, 3\right), \left(-1-\sqrt{13}, 3\right)$
Asymptotes $y = \pm\frac{2}{3}(x+1)+3$



7. $\dfrac{(y+2)^2}{16} - \dfrac{(x-5)^2}{20} = 1$
Center $(5,-2)$
Transverse axis on $x = 5$
Conjugate axis on $y = -2$
Vertices $(5,2), (5,-6)$
Foci $(5,4), (5,-8)$
Asymptotes $y = \pm\frac{2\sqrt{5}}{5}(x-5)-2$



8. $\dfrac{(x-4)^2}{8} - \dfrac{(y-2)^2}{18} = 1$
Center $(4,2)$
Transverse axis on $y=2$
Conjugate axis on $x=4$
Vertices $\left(4+2\sqrt{2},2\right), \left(4-2\sqrt{2},2\right)$
Foci $\left(4+\sqrt{26},2\right), \left(4-\sqrt{26},2\right)$
Asymptotes $y = \pm\frac{3}{2}(x-4)+2$



9. $\dfrac{x^2}{3} - \dfrac{(y-5)^2}{12} = 1$
Center $(0,5)$
Transverse axis on $y=5$
Conjugate axis on $x=0$
Vertices $(\sqrt{3},5), (-\sqrt{3},5)$
Foci $(\sqrt{15},5), (-\sqrt{15},5)$
Asymptotes $y = \pm 2x+5$

10. $\dfrac{(y+2)^2}{5} - \dfrac{(x-3)^2}{18} = 1$
Center $(3,-2)$
Transverse axis on $x=3$
Conjugate axis on $y=-2$
Vertices $(3,-2+\sqrt{5}), (3,-2-\sqrt{5})$
Foci $(3,-2+\sqrt{23}), (3,-2-\sqrt{23})$
Asymptotes $y = \pm\frac{\sqrt{10}}{6}(x-3)-2$

11. $\dfrac{(x-3)^2}{25} - \dfrac{(y+1)^2}{9} = 1$
Center $(3,-1)$
Transverse axis on $y=-1$
Conjugate axis on $x=3$
Vertices $(8,-1), (-2,-1)$
Foci $\left(3+\sqrt{34},-1\right), \left(3-\sqrt{34},-1\right)$
Asymptotes $y = \pm\frac{3}{5}(x-3)-1$

12. $\dfrac{(y+4)^2}{6} - \dfrac{(x+2)^2}{5} = 1$
Center $(-2,-4)$
Transverse axis on $x=-2$
Conjugate axis on $y=-4$
Vertices $\left(-2,-4+\sqrt{6}\right), \left(-2,-4-\sqrt{6}\right)$
Foci $\left(-2,-4+\sqrt{11}\right), \left(-2,-4-\sqrt{11}\right)$
Asymptotes $y = \pm\frac{\sqrt{30}}{5}(x+2)-4$

13. $\dfrac{(y-7)^2}{16} - \dfrac{(x-3)^2}{9} = 1$

14. $\dfrac{(x-4)^2}{16} - \dfrac{(y-1)^2}{33} = 1$

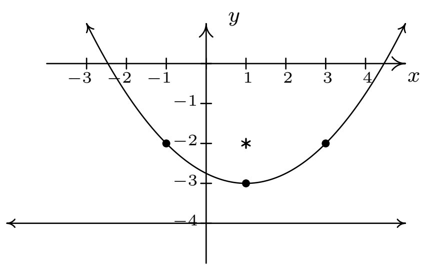
15. $\dfrac{y^2}{25} - \dfrac{x^2}{39} = 1$

16. $\dfrac{x^2}{16} - \dfrac{y^2}{9} = 1$

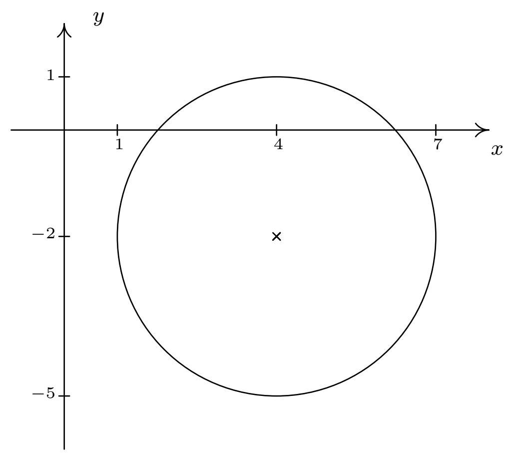
17. $\dfrac{(x-8)^2}{25} - \dfrac{(y-2)^2}{4} = 1$

18. $\dfrac{(x-6)^2}{256} - \dfrac{(y-5)^2}{64} = 1$

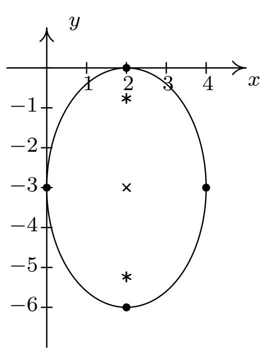
19. $(x-1)^2 = 4(y+3)$



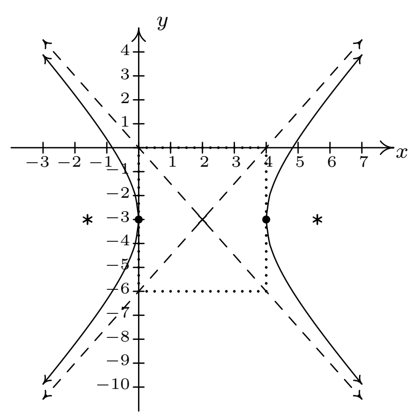
20. $(x-4)^2 + (y+2)^2 = 9$



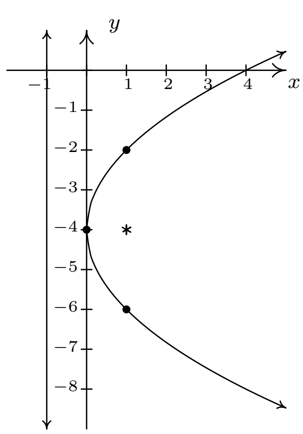
21. $\dfrac{(x-2)^2}{4} + \dfrac{(y+3)^2}{9} = 1$



22. $\dfrac{(x-2)^2}{4} - \dfrac{(y+3)^2}{9} = 1$



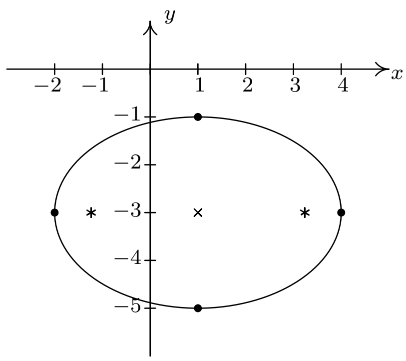
23. $(y+4)^2 = 4x$



24. $\dfrac{(x-1)^2}{1} + \dfrac{y^2}{4} = 0$

The graph is the point $(1,0)$ only.

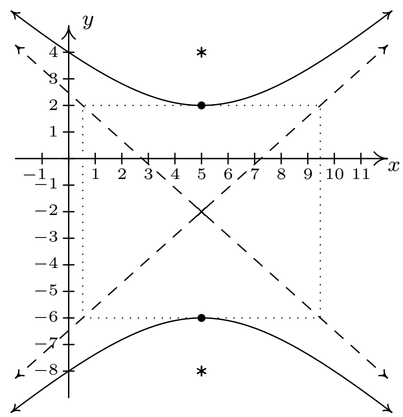
25. $\dfrac{(x-1)^2}{9} + \dfrac{(y+3)^2}{4} = 1$



26. $(x-3)^2 + (y+2)^2 = -1$
There is no graph.

27. $\dfrac{(x+3)^2}{2} + \dfrac{(y-1)^2}{1} = -\dfrac{3}{4}$
There is no graph.

28. $\dfrac{(y+2)^2}{16} - \dfrac{(x-5)^2}{20} = 1$



30. By placing Station A at $(0,-50)$ and Station B at $(0,50)$, the two second time difference yields the hyperbola $\frac{y^2}{36} - \frac{x^2}{2464} = 1$ with foci A and B and center $(0,0)$. Placing Station C at $(-150,-50)$ and using foci A and C gives us a center of $(-75,-50)$ and the hyperbola $\frac{(x+75)^2}{225} - \frac{(y+50)^2}{5400} = 1$. The point of intersection of these two hyperbolas which is closer to A than B and closer to A than C is $(-57.8444, -9.21336)$ so that is the epicenter.

31. (b) $\dfrac{x^2}{9} - \dfrac{y^2}{27} = 1$.

32. The tower may be modeled (approximately)[12] by $\frac{x^2}{12100} - \frac{(y-330)^2}{34203} = 1$. To find the height, we plug in $x = 137.5$ which yields $y \approx 191$ or $y \approx 469$. Since the top of the tower is above the narrowest point, we get the tower is approximately 469 feet tall.

[12] The exact value underneath $(y-330)^2$ is $\frac{52707600}{1541}$ in case you need more precision.